Financial Enterprise Risk Management

Chapter 9: Some Useful Statistics (Background Only)

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Key Exam Topics in This Lesson

Basic Statistics

- Basic Sample Population Statistics
- Skew and Kurtosis
- Normal Distribution (a.k.a. Elliptical or Gaussian)
- Typical Risk Distribution
- Types of Correlation Measures
- BA II Plus Calculator Practice!

Basic Sample Population Statistics

For a univariate sample of T observations:

$$\text{Sample Mean} = \overline{X} = \frac{1}{T}\sum_{t=1}^{T} X_t$$

$$\text{Median} = 50\text{th percentile}$$

$$\text{Mode} = \text{the most common observation}$$

$$\text{Sample Variance} = s^2 = \frac{1}{T-1}\sum_{t=1}^{T}(X_t - \overline{X})^2$$

$$\text{Range} = \max X_t - \min X_t$$

For a 2-variable sample (correlation and covariance):

$$r_{X,Y} = \frac{s_{X,Y}}{s_X s_Y} \qquad s_{X,Y} = \frac{1}{T-1}\sum_{t=1}^{T}\left(X_t - \overline{X}\right)\left(Y_t - \overline{Y}\right)$$

Skew and Kurtosis

Skew = third central moment, normalized

$$\omega = \frac{1}{T}\left(\frac{\sum_{t=1}^{T}(X_t - \mu)^3}{\sigma^3}\right) \qquad \text{(Population)}$$

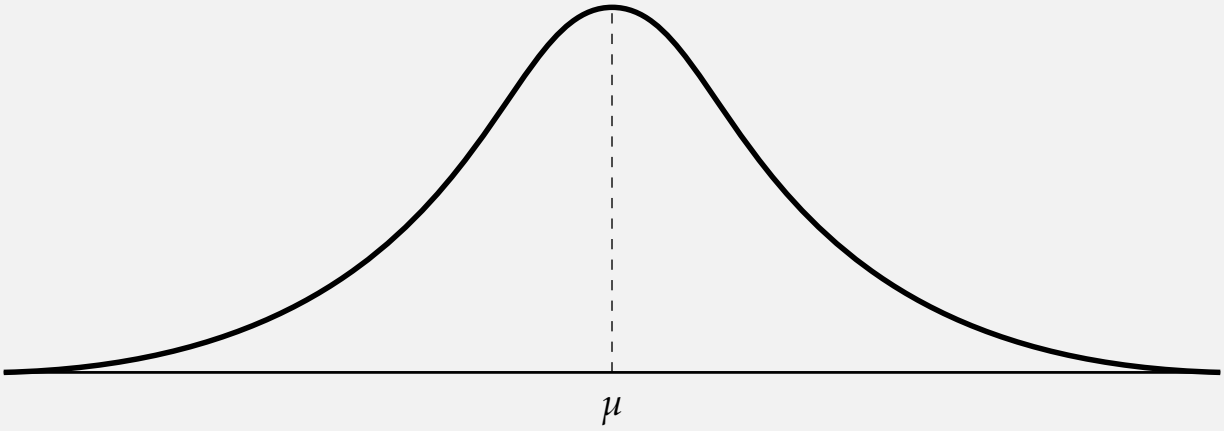
$$w = \left(\frac{T}{(T-1)(T-2)}\right)\left(\frac{\sum_{t=1}^{T}\left(X_t - \overline{X}\right)^3}{s^3}\right) \qquad \text{(Sample)}$$

Kurtosis = fourth central moment, normalized against Gaussian distribution

$$\kappa = \frac{1}{T}\frac{\sum_{t=1}^{T}\left(X_t - \mu\right)^4}{\sigma^4} - 3 \qquad \text{(Population)}$$

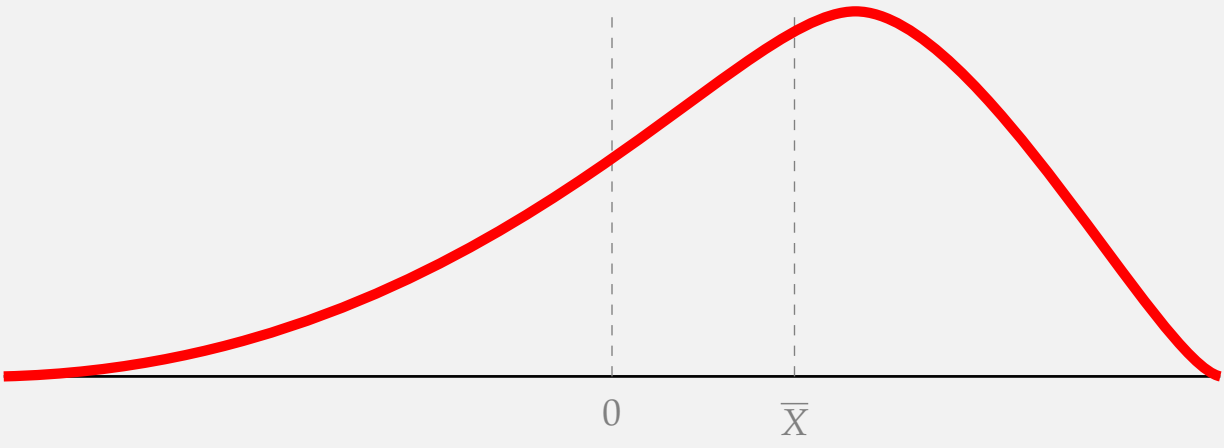
$$k = \left(\frac{T(T+1)}{(T-1)(T-2)(T-3)}\right)\left(\frac{\sum_{t=1}^{T}\left(X_t - \overline{X}\right)^4}{s^4}\right) - \frac{3(T-1)^2}{(T-2)(T-3)} \qquad \text{(Sample)}$$

Normal Distribution (a.k.a. Elliptical or Gaussian)



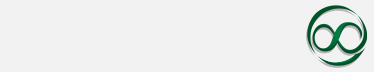
- Mean = mode = median
- Skew = 0
- Kurtosis = 3 (“mesokurtic”)

Typical Risk Distribution



- Skew < 0: long left tail $\Rightarrow$ many more worse-than-expected results
- Kurtosis > 3 (“leptokurtic”): fatter tails $\Rightarrow$ higher probability of extreme results

Types of Correlation Measures



1. **Pearson's rho** – a.k.a. linear correlation

$$\rho = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y} \qquad r_{X,Y} = \frac{s_{X,Y}}{s_X s_Y}$$

2. **Spearman correlation** = Pearson linear correlation of the observations' ranks

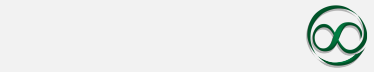
$$\rho_s = \rho[\text{Ranks of } X, \text{Ranks of } Y]$$

3. **Kendall's tau** – measures the observations' tendency to move together

$$\tau = \frac{\text{Concordant Pairs} - \text{Discordant Pairs}}{\text{Total Possible Pairs}}$$

4. **Tail correlation** – correlation of tail values only

BA II Plus Calculator Practice!



Suppose we have 5 observations of X and Y

t	X_t	Y_t
1	10	20
2	95	25
3	15	10
4	35	15
5	45	30

Calculate the following statistics using your BA II Plus calculator:

1. Sample mean of X and Y
2. Sample standard deviation of X and Y
3. Covariance of X and Y
4. Linear correlation of X and Y

Pause Video

No peeking!

BA II Plus Calculator Practice!

See video for a walk through of the BA II Plus's statistical functions!

t	X_t	Y_t
1	10	20
2	95	25
3	15	10
4	35	15
5	45	30
Sample mean	40.00	20.00
s	33.91	7.91

$$s_{X,Y} = \frac{1}{4}\sum_{t=1}^{5}(X_t - 40)(Y_t - 20) = 150$$

$$r_{X,Y} = \frac{150}{33.91 \times 7.91} = 55.92\% = \text{Pearson's rho}$$