ILA Life Finance & Valuation Exam (US) – Spring 2015

Question 10

Video By: J. Eddie Smith, IV, FSA, MAAA

Parts Covered in This Video

Q10(a) – Mean GAAP Reserves

Q10(b) – Pre-Tax Stat and GAAP Income

Q10(a) – Terminal Reserve Formulas

Terminal reserve formulas for maintenance expense reserve (MER) and DAC:

$$
\begin{aligned}
MER_t &= PVME_t - k^{ME} \times PVGP_t \\
&= \frac{Surv_{1-1}(MER_{t-1} - ME_t + k^{ME} \times GP_t)(1+i)}{Surv_t} \\
k^{ME} &= \frac{PVME_0}{PVGP_0} \\
DAC_t &= k^{DAC} \times PVGP_t - PVDAE_t \\
&= \frac{Surv_{1-1}(DAC_{t-1} + DAE_t - k^{DAC} \times GP_t)(1+i)}{Surv_t} \\
k^{DAC} &= \frac{PVDAE_0}{PVGP_0}
\end{aligned}
$$

Q10(a) – Mean Reserve Formulas

$$
\begin{aligned}
\text{Mean MER}_t &= (1-h) \times (MER_{t-1} - ME_t + k^{ME} \times GP_t) + h \times MER_t \\
\text{Mean DAC}_t &= (1-h) \times (DAC_{t-1} + DAE_t - k^{DAC} \times GP_t) + h \times DAC_t
\end{aligned}
$$

where $h = 0.5$ for this problem since we are exactly in the middle of the policy year

Q10(a) – Discount and Survival Factors

The problem gave interest, mortality, and lapse assumptions

We're going to be using these a lot, so let's program them in the ole calculator...

$$
\begin{aligned}
v &= 1.05^{-1} = 0.9524 \\
Surv_t &= Surv_{t-1}(1 - q_t)(1 - w_t) \\
Surv_0 &= 1.0000 \\
Surv_1 &= Surv_0(1 - 0.620/1000)(1 - 0.15) = 0.8495 \\
Surv_2 &= Surv_1(1 - 0.655/1000)(1 - 0.05) = 0.8065
\end{aligned}
$$

You will get slightly different answers depending on how much you round these

Q10(a) – PV of Gross Premiums At Issue

Let's calculate the PV of GPs first:

$$PVGP_0 = 100 \times (1.32 + 1.35 \times Surv_1 \times v + 1.39 \times Surv_2 \times v^2) = 342.90$$

Q10(a) – Maintenance Reserve Calculations

There are 3 maintenance expenses given in the problem:

1. Maintenance expense per policy = 30

$$PV_0 = 30(1 + Surv_1 \times v + Surv_2 \times v^2) = 76.22$$

2. Ultimate commission = 5% of premium

$$PV_0 = 0.05 \times PVGP_0 = 17.14$$

3. Premium tax = 2% of premium

$$PV_0 = 0.02 \times PVGP_0 = 6.86$$

Total PVME = 100.22 and $k^{ME} = \frac{100.22}{342.90} = 0.29227$

Q10(a) – Maintenance Reserve Calculations (Continued)

We know $MER_0 = 0$ (always), but we need MER_1, which is easiest to get retrospectively

$$\begin{aligned} MER_1 &= \frac{Surv_0(MER_0 - ME_1 + k^{ME} \times GP_1)(1+i)}{Surv_1} \\ &= \frac{1.000\big(0 - (30 + 0.07 \times 132) + 0.29227 \times 132\big)(1.05)}{0.8495} \\ &= \frac{1.000(-0.66)(1.05)}{0.8495} \\ &= -0.82 \end{aligned}$$

Shortcut: ignore % of premium expenses and use $k^{ME} = \frac{76.22}{342.90} = 0.22227$:

$$\begin{aligned} MER_1 &= \frac{1.000\big(0 - 30 + 0.22227 \times 132\big)(1.05)}{0.8495} \\ &= \frac{1.000(-0.66)(1.05)}{0.8495} \\ &= -0.82 = \text{correct MER, but understates expense ratio} \end{aligned}$$

Q10(a) – Mean Maintenance Reserve

$$\begin{aligned}\text{Mean MER}_t &= (1-h) \times (MER_{t-1} - ME_t + k^{ME} \times GP_t) + h \times MER_t \\ &= 0.5 \times (-0.66) + 0.5 \times (-0.82) \\ &= -0.74\end{aligned}$$

Q10(a) – Mean DAC

DAE = commissions in excess of ultimate = $50\% - 5\% = 45\%$

$$k^{DAC} = \frac{0.45 \times 100 \times 1.32}{342.90} = \frac{59.40}{342.90} = 0.17323$$

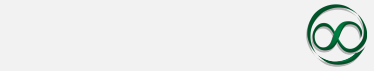
$$DAC_t = \frac{Surv_{1-1}(DAC_{t-1} + DAE_t - k^{DAC} \times GP_t)(1+i)}{Surv_t}$$

$$DAC_0 = 0$$

$$\begin{aligned}DAC_1 &= \frac{1.0000(0 + 59.40 - 0.17323 \times 132)(1.05)}{0.8495} \\ &= \frac{1.0000(36.53)(1.05)}{0.8495} = 45.16\end{aligned}$$

$$\begin{aligned}\text{Mean DAC} &= 0.5 \times (36.53) + 0.5 \times (45.16) \\ &= 40.85\end{aligned}$$

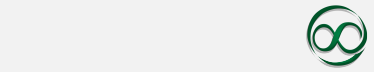
ILA LFV-U Exam – Spring 2015 – Q10



Q10(a) – Mean GAAP Reserves

Q10(b) – Pre-Tax Stat and GAAP Income

Q10(b) – Profit Formulas



$$
\begin{aligned}
StatProfit_t &= ProdCF_t + StatInvInc_t - StatResIncr_t \\
ProdCF_t &= \underbrace{Prem_t - Exp_t}_{BOYProdCF_t} - Ben_t \\
StatInvInc_t &= (BOYProdCF_t + StatRes_{t-1}) \times i_{stat}
\end{aligned}
$$

$$
\begin{aligned}
GAAPProfit_t &= ProdCF_t + GAAPInvInc_t - GAAPNetLiabIncr_t \\
GAAPInvInc_t &= (BOYProdCF_t + GAAPNetLiab_{t-1}) \times i_{gaap} \\
GAAPNetLiab_t &= BenRes_t + MER_t - DAC_t \text{ (before survivorship)} \\
GAAPNetLiabIncr_t &= Surv_t(GAAPNetLiab_t) - Surv_{t-1}(GAAPNetLiab_{t-1})
\end{aligned}
$$

Q10(b) – Stat Profit Calculation

$$
\begin{aligned}
ProdCF_1 &= 132 - (0.52 \times 132 + 30) - (0.62 \times 100) \\
&= 33.36 - 62.00 = -28.64 \\
StatInvInc_1 &= (0 + 33.36) \times 0.05 = 1.67 \\
StatProfit_1 &= -28.64 + 1.67 - (0 - 0) \\
&= \boxed{-26.97}
\end{aligned}
$$

Q10(b) – GAAP Profit Calculation

$$
\begin{aligned}
ProdCF_1 &= -28.64 \\
GAAPInvInc_1 &= (0 + 33.36) \times 0.05 = 1.67 \\
&= StatInvInc_1 \text{ since both liabilities are zero at time } 0 \\
&\neq StatInvInc_t \text{ for } t > 1 \\
GAAPNetLiab_0 &= 0 \\
GAAPNetLiab_1 &= BenRes_1 + MER_1 - DAC_1 \\
&= 2.16 + (-0.82) - 45.16 = -43.82 \\
GAAPNetLiabIncr_1 &= 0.8495(-43.82) - 1.0000(0) = -37.22 \\
GAAPProfit_1 &= -28.64 + 1.67 - (-37.22) \\
&= \boxed{10.25}
\end{aligned}
$$

Q10(b) – GAAP Profit Calculation – Alternative

We were given $BenRes_1 = 2.16$ and told experience = GAAP assumptions

We can solve for the year 1 benefit premium using a retrospective reserve:

$$2.16 = \frac{(0 + BenPrem_1)(1.05) - 0.62 \times 100}{0.8495}$$

$$BenPrem_1 = 60.7951$$

This means that the benefit ratio is

$$BenRatio = \frac{60.7951}{132} = 0.4606$$

We can now solve for the profit margin:

$$\begin{aligned} ProfitMargin &= 1 - BenRatio - k^{ME} - k^{DAC} \\ &= 1 - 0.4606 - 0.2922 - 0.1732 = 7.4\% \\ GAAPProfit_1 &= (ProfitMargin \times GP_1)(1+i) = (0.074 \times 132)(1.05) \\ &= \boxed{10.25} \end{aligned}$$