FPT Method

FPT = Modified NLP Method with a formulaic expense allowance

$$ {}_tV_x^{FPT} = {}_tV_x^{NLP} - {}_tVE_x = \text{PVFB}_t - \underbrace{(\text{PVPB}_t + \text{PVPE}_t)}_{\text{PVNP}_t} $$

$$ {}_tVE_x = \text{PVPE}_t = PE_0 \times \ddot{a}_{x+t} $$

$$ PE_0 = \frac{EA_x}{\ddot{a}_x} $$

$$ EA_x = NP_1 - c_x = \left(\frac{\text{PVFB}_1}{\ddot{a}_{x+1}}\right) - c_x $$

$$ c_x = v \cdot q_x \cdot \text{DB} = \text{first-year cost of insurance} $$

$$ NP_t = \begin{cases} c_x & \text{for } t = 0 \quad (\alpha) \\ PB_t + PE_t = \dfrac{\text{PVFB}_1}{\ddot{a}_{x+1}} \cdot r_t^{GP} & \text{for } t \geq 1 \quad (\beta) \end{cases} $$

EA $\neq$ actual expenses

$${}_0V_x^{FPT} = {}_1V_x^{FPT} = 0$$

$${}_tV_x^{FPT} \leq {}_tV_x^{NLP}$$

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Example Contract (5-Year Term)

Assumptions:

- Issue age = 55
- Death benefit = 100,000
- Benefit period = 5 years
- Premium paying period = 5 years
- Gross premium per 1000 = 9
- Valuation interest rate = 5%

t	x	q_{55+t}
0	55	0.0053
1	56	0.0064
2	57	0.0077
3	58	0.0090
4	59	0.0101

Task 1: Determine NLP Reserve

Determine the NLP reserve for all policy years the contract is in force.

Task 1 Solution

For all years:

$$ {}_tV_{55}^{NLP} = \text{PVFB}_t - NP_0 \cdot \ddot{a}_{55+t:\overline{5-t}|} $$
$$ \text{PVFB}_t = 100{,}000 A_{55+t:\overline{5-t}|} $$

Since the GP is level, $NP_t = NP_0$:

$$ \begin{aligned} NP_0 &= \left(\frac{\text{PVFB}_0}{\ddot{a}_{55:\overline{5}|}} \right) \\ &= \frac{3234.86}{4.4905} \\ &= 720.37 \end{aligned} $$

t	x	q_{55+t}	${}_tp_{55}$	PVFB_t	$\ddot{a}_{55+t:\overline{5-t}\|}$
0	55	0.0053	1.0000	3,234.86	4.4905
1	56	0.0064	0.9947	2,881.88	3.6846
2	57	0.0077	0.9883	2,401.34	2.8370
3	58	0.0090	0.9807	1,765.00	1.9438
4	59	0.0101	0.9719	961.90	1.0000

Year 3 sample calculations (other ways possible too!):

$$ \begin{aligned} \text{PVFB}_3 &= 100{,}000 \left[\frac{\frac{0.0090(0.9807)}{1.05} + \frac{0.0101(0.9719)}{1.05^2}}{0.9807} \right] \\ &= 1{,}765 \end{aligned} $$

$$ \ddot{a}_{58:\overline{2}|} = \frac{0.9807 + \frac{0.9719}{1.05}}{0.9807} = 1.94 $$

Rounding differences within a few cents are likely and not important.

Task 1 Solution Summary

t	x	PVFB_t	$-$	NP_0	$\times$	$\ddot{a}_{55+t:\overline{5-t}\|}$	$=$	${}_tV_{55}^{NLP}$
0	55	3,234.86	$-$	720.37	$\times$	4.4905	$=$	0.00
1	56	2,881.88	$-$	720.37	$\times$	3.6846	$=$	227.60
2	57	2,401.34	$-$	720.37	$\times$	2.8370	$=$	357.65
3	58	1,765.00	$-$	720.37	$\times$	1.9438	$=$	364.73
4	59	961.90	$-$	720.37	$\times$	1.0000	$=$	241.53
5	60							0.00

Retrospective approach:

$$ {}_1V_{55}^{NLP} = \frac{(0 + 720.37)(1.05) - 0.0053(100{,}000)}{1 - 0.0053} = 227.60 $$
$$ {}_2V_{55}^{NLP} = \frac{(227.60 + 720.37)(1.05) - 0.0064(100{,}000)}{1 - 0.0064} = 357.65 $$

etc.

Task 2: Determine FPT Reserve

Determine the FPT reserve for the same contract.

Task 2 Solution – Getting the Net Premiums

For all years:

$$ {}_tV^{FPT}_{55} = {}_tV^{NLP}_{55} - {}_tVE_{55} = {}_tV^{NLP}_{55} - PE_0\ddot{a}_{55+t} $$

${}_tVE_x$ = PV of PEs, which we need the EA to determine:

$$ \begin{aligned} EA_{55} &= \left(\frac{\text{PVFB}_1}{\ddot{a}_{56:\overline{4}|}}\right) - c_{55} \\ &= \frac{2881.88}{3.6846} - \frac{0.0053(100,000)}{1.05} \\ &= \underbrace{782.14}_{NP_1} - \underbrace{504.76}_{c_{55}} = 277.38 \end{aligned} $$

$$ PE_0 = \frac{EA_{55}}{\ddot{a}_{55:\overline{5}|}} = \frac{277.38}{4.4905} = 61.77 $$

$$ NP_1 = PB_0 + PE_0 = 720.37 + 61.77 = 782.14 = PB_0 \text{ for age } \underline{56} $$

Task 2 Solution: Unamortized EA

The unamortized EA is essentially a reserve for (formulaic) costs recognized at issue

t	x	EA_{55}	$-$	PE_0	$\times$	$\ddot{a}_{55+t:\overline{5-t}\rceil}$	$=$	${}_tVE_{55}$
0	55	277.38	$-$	61.77	$\times$	4.4905	$=$	0.00
1	56	0.00	$-$	61.77	$\times$	3.6846	$=$	-227.60
2	57	0.00	$-$	61.77	$\times$	2.8370	$=$	-175.24
3	58	0.00	$-$	61.77	$\times$	1.9438	$=$	-120.07
4	59	0.00	$-$	61.77	$\times$	1.0000	$=$	-61.77
5	60						$=$	0.00

Task 2 Solution: Relationship Between NLP and FPT Reserve

Same as NLP

t	x	${}_tV^{NLP}_{55}$	$-$	${}_tVE_{55}$	$=$	${}_tV^{FPT}_{55}$	$=$	PVFB_t	$-$	(PB_0+PE_0)	$\times$	$\ddot{a}_{55+t:\overline{5-t}\rceil}$
0	55	0.00	$-$	0.00	$=$	0.00						
1	56	227.60	$-$	227.60	$=$	0.00	$=$	2,881.88	$-$	(720.37 + 61.77)	$\times$	3.6846
2	57	357.65	$-$	175.24	$=$	182.41	$=$	2,401.34	$-$	(720.37 + 61.77)	$\times$	2.8370
3	58	364.73	$-$	120.07	$=$	244.67	$=$	1,765.00	$-$	(720.37 + 61.77)	$\times$	1.9438
4	59	241.53	$-$	61.77	$=$	179.76	$=$	961.90	$-$	(720.37 + 61.77)	$\times$	1.0000
5	60	0.00			$=$	0.00						

In the first policy year, the FPT $NP_0 = \alpha = c_x$

$$\begin{aligned}
{}_0V^{FPT}_{55} &= \text{PVFB}_0 - \text{PVNP}_0 \\
&= \text{PVFB}_0 - (c_{55} + v_1 p_{55}\text{PVNP}_1) \\
&= 3234.86 - \left(504.76 + \frac{0.9947}{1.05}(782.14 \times 3.6846)\right) \\
&= 0
\end{aligned}$$

Task 2 Solution: Retrospective View

$$
\begin{aligned}
{}_0V^{FPT}_{55} &= 0 \\
{}_1V^{FPT}_{55} &= \frac{(0+c_{55})(1.05)-100{,}000(q_{55})}{1-q_{55}} \\
&= \frac{(0+100{,}000vq_{55})(1.05)-100{,}000(q_{55})}{1-q_{55}} \\
&= 0 \\
{}_2V^{FPT}_{55} &= \frac{(0+782.14)(1.05)-100{,}000(0.0064)}{1-0.0064} \\
&= 182.41
\end{aligned}
$$

etc.

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