THE INFINITE ACTUARY'S

SAMPLE DETAILED STUDY GUIDE FOR THE

QFI Quant Exam

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MFD Chapter 1: Financial Derivatives – A Brief Introduction

Neftci

Overview of This Reading

Welcome to the QFI Quant seminar! The syllabus starts off by going through the MFD textbook. The first chapter will likely be straightforward for most students, as it reviews the basics of financial instruments and derivatives.

Key topics for the exam include:

- List and describe the types of derivatives
- Compare futures vs forward contracts
- Explain the meaning of "in-the-money" and "out-of-the-money"

Definition of Derivatives

- **Practitioner Definition of Derivatives:** Securities that derive their value from cash market instruments[1]
- **Ingersoll's Definition of Derivatives:** A financial contract is a derivative security, or a contingent claim, if its value at expiration date T is determined by the market price of the underlying cash instrument at time T

Types of Derivatives

- Futures and forwards
- Options
- Swaps

Five Main Groups of Underlying Assets

- Stocks
- Currencies
- Interest rates
- Indexes
- Commodities

[1] The cash market is a public financial market in which financial instruments or commodities are traded for immediate delivery. It contrasts with the futures market, in which delivery is due at a later date.

Cash-and-Carry Markets

- Examples: gold, silver, currencies, T-bonds
- Can buy a forward contract or borrow money to buy gold and hold until expiration
- By no arbitrage, can use the cash-and-carry strategy to determine forward prices
- Any relevant information concerning future supplies and demands of the underlying instrument is expected to make the cash price and the future price change by the same amount

Price-Discovery Markets

- Price-Discovery Markets are markets where it is physically impossible to hold the good to expiration (e.g. spring wheat futures sold the prior fall)
- Arbitrage cannot be applied in the same sense as the cash-and-carry strategy, but can look at market price to determine information about the future supply and demand of the underlying

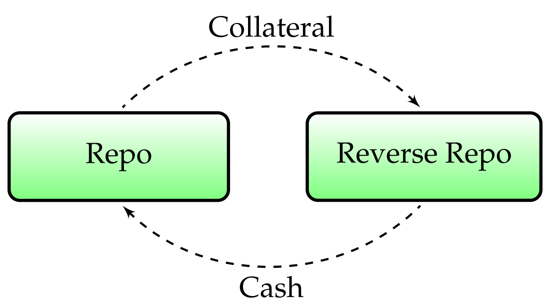
Futures and Forwards

- A *forward contract* is an obligation to buy (sell) an underlying asset at a specified forward price on a known date
- If at expiration the cash price is higher than the forward price, the long position makes a profit
- The table below summarizes the key differences between future and forward contracts:

Characteristic	Futures	Forwards
Mark to Market	Yes	No
Settlement	Daily	Upon Maturity
Margin System	Yes	No
Trading	Exchanges	OTC
Types	Standardized	Customized
Contract Flexibility	Less	More
Liquidity	More	Less
Counterparty Risk	Less	More

Repurchase Agreements (Repos and Reverse Repos)

- A *repo* is a transaction in which one party sells securities to another party in return for cash, with an agreement to repurchase equivalent securities at an agreed price and on an agreed future date.
- Difference between repo price and original sale price gives the *repo rate*
- Usually used to raise short-term capital
- Classified as a money market instrument



- Three types of repos:
  1. Overnight - one-day maturity transaction
  2. Term - a repo with a specific end date
  3. Open - has no end date
- A *flexible repo* is a repo with a flexible withdrawal schedule
  - The party who bought the security could sell it partially at times before and at maturity, as opposed to just at maturity
  - In a *secured flexible repo*, the municipality receives collateral. In an *unsecured flexible repo*, the customer does not receive collateral – and the customer typically receives a higher spread to compensate for this extra risk

Options

- A *European-type call option* on a security S_t is the right to buy the security at a strike price K. This right may be exercised at the expiration date T of the option. The call option can be purchased for a price of C_t, called the premium, at time $t < T$
- American options can be exercised any time between writing and expiration
- Futures and forwards are an obligation at expiration, while options give the right but not the obligation
- $C_T = \max(S_T - K, 0)$. Note that this is a non-linear payoff function:
  - Expires *out-of-the-money* when $S_T < K$
  - Expires *in-the-money* when $S_T > K$

Swaps

- Exchange of one set of cash flows for another
- A *swap* is the simultaneous selling and purchasing of cash flows involving currencies, interest rates, and a number of other financial assets
- It is always possible to decompose simple swap deals into a basket of simpler forward contracts. Swaps can then be priced through forwards
- Cancelable swaps
  - Swaps where one or both parties have the right but not the obligation to cancel the swap before maturity.
  - For example, a callable swap allows the payer of the fixed rate to cancel the swap as long as the present value of future payments is paid.
  - Can be used for hedging if the asset duration need to be shortened.
  - Also popular with institutions who have obligations that they can repay before maturity, such as callable bonds

Conclusion

- This reading explores the basics of common financial instruments
- It is important to have a solid foundation on these topics, because we will revisit many of these instruments (i.e. looking at pricing) through the QFI Quant syllabus

FAQ Q23: Jensen's Inequality

Paul Wilmott (2009)

Overview of This Reading

Next, the syllabus goes through a series of short FAQ readings, each reading is typically just a few pages. The first topic is Jensen's Inequality.

Key topics for the exam include:

- Define convex functions
- State Jensen's Inequality
- Explain the importance of Jensen's Inequality in financial markets

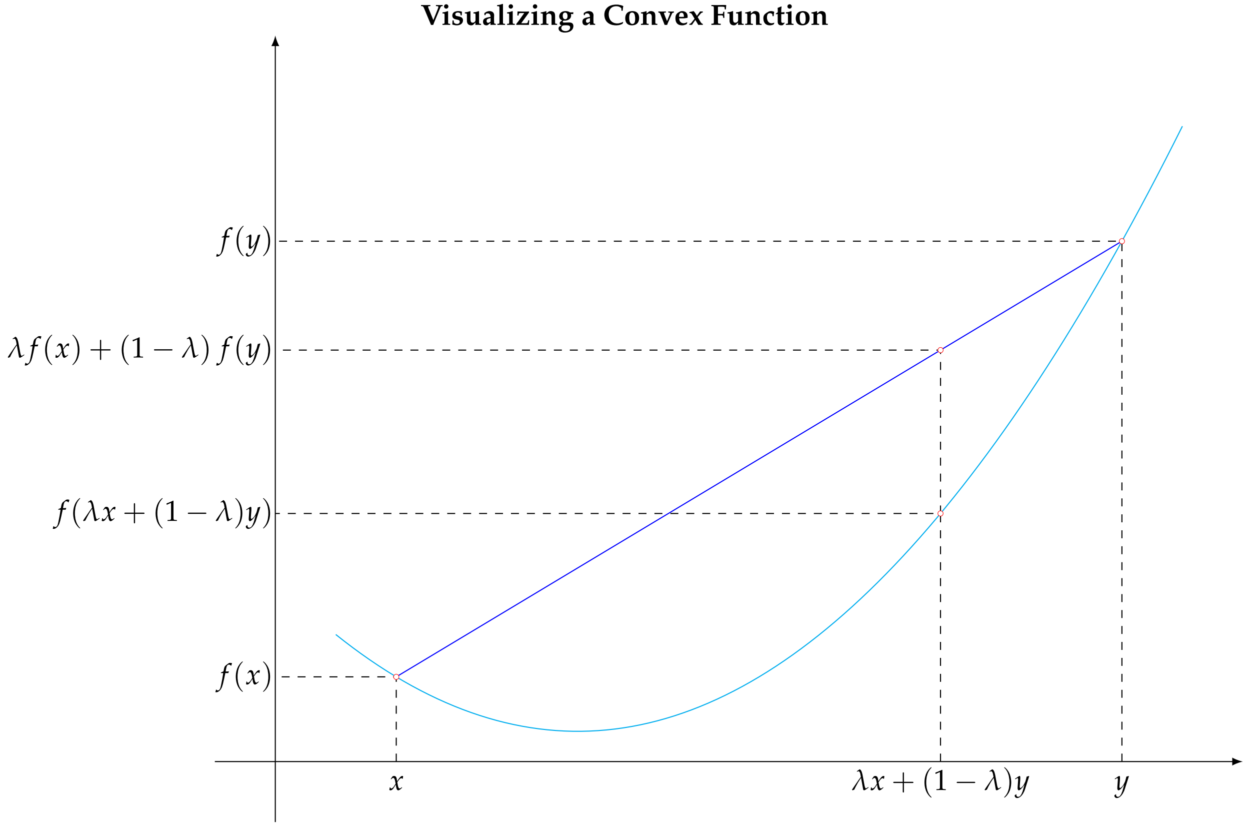
Definition of Convexity

A function f is convex on an interval if for every x and y in that interval

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$$

for any $0 \leq \lambda \leq 1$.

Visualizing a Convex Function



- Convex functions:
  - All linear functions
  - $y = x^2, y = x^4$
  - Payoff and profit functions for long calls/puts
- Not convex functions:
  - $y = \sqrt{x}$
  - $y = -x^2$

Jensen's Inequality

If f is a convex function and x is a random variable then

$$E(f(x)) \geq f(E(x))$$

- Importance in Finance:
  - Jensen's Inequality gives insight into why non-linear instruments such as options have inherent value
  - Whenever a contract has convexity in a variable or parameter, and that variable is random, then an allowance must be made for this in pricing

Approximating Convexity

Jensen's inequality states that $E(f(x)) \geq f(E(x))$, but it is helpful to know by approximately how much the left-hand-side is greater than the right-hand-side.

- $E(f(x)) \geq f(E(x)) \Rightarrow E(f(S)) = f(E(S)) +$ Convexity
- Let $\bar{S} = E(S)$ and $S = \bar{S} + \epsilon$
- Given the above definition, it is clear that $E(\epsilon) = 0$
- Then $E(f(S))$

  $= E(f(\bar{S} + \epsilon))$

  $= E\left[f(\bar{S}) + \epsilon f'(\bar{S}) + \frac{1}{2}\epsilon^2 f''(\bar{S}) + \dots\right]$

  $\approx E\left[f(\bar{S}) + \epsilon f'(\bar{S}) + \frac{1}{2}\epsilon^2 f''(\bar{S})\right]$

  $= f(\bar{S}) + E(\epsilon f'(\bar{S})) + E(\frac{1}{2}\epsilon^2 f''(\bar{S}))$

  $= f(\bar{S}) + \frac{1}{2}E(\epsilon^2) f''(\bar{S})$

  $= f(E(S)) +$ Convexity

- So we have that $E(f(S)) = f(E(S)) + \text{Convexity}$, where convexity is approximated as:

$$\boxed{\text{Convexity } = \frac{1}{2} \cdot \underbrace{E(\epsilon^2)}_{\text{randomness}} \cdot \underbrace{f''(\bar{S})}_{\text{function convexity}}}$$

- The equation above solidifies the following point that we saw before: whenever a contract has convexity in a variable or parameter, and that variable is random, then an allowance must be made for this in pricing

Example: Rolling a Die

- Suppose you roll a fair die once. The result of the roll is squared, so that you receive x^2 dollars
- Note that $f(x) = x^2$ is a convex function
- The expected roll is given by: $E(X) = \frac{1+2+3+4+5+6}{6} = 3.5$
- The expected dollars received is given by: $E(X^2) = \frac{1^2+2^2+3^2+4^2+5^2+6^2}{6} = 15\frac{1}{6}$
- Summarizing, we have that:
  - $f(x) = x^2$ is a convex function, so Jensen's inequality applies
  - $f(E(X)) = 3.5^2 = 12.25 < E(f(X)) = E(X^2) = 15\frac{1}{6}$
- Actually, let's move past the die example, but think about the same convex function $f(x) = x^2$. Applying Jensen's inequality gives that $f(E(X)) \leq E(f(x))$, or $E(X)^2 \leq E(X^2)$
  - Rearranging, this gives the familiar result that variance is non-negative:
  - $V(X) = E(X^2) - E(X)^2 \geq 0$

What about Concave Functions? (Bonus Section)

- *I wanted to add a short extra section here not in the reading, but this is useful and I recommend reading through this!*
- We have looked at a relevant inequality for convex functions, but what if we have a concave function like natural log? Is there a relevant inequality we can use for concave functions?
- The answer is yes, we simply switch the direction of the inequality[2]
- For a convex function f, we have that $E(f(x)) \geq f(E(x))$
  - This is Jensen's inequality, which we have already seen on the prior pages of this reading
  - Example $E(X^2) \geq E(X)^2$ where $f(x) = x^2$ is a convex function
- For a concave function f, we have that $f(E(x)) \geq E(f(x))$
  - This second statement is now about concave functions (instead of convex functions) and we simply flip the direction of the inequality
  - Example: $\ln(E(X)) \geq E(\ln(X))$ where $f(x) = \ln(x)$ is a concave function

[2]We will not go through a formal proof, but the reasoning is straightforward and a result of the fact that a function g is convex if and only if $-g$ is concave. Put simply, this negative sign results in the inequality being flipped.

